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Tax Evasion Modeling under Penalties and Refusal of Banks to Grant Loan

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Final Report

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The current paper is a final report of the Project: **Tax evasion modeling under penalties and loan refusal** (№99-306).

We express our gratitude to the EERC experts for that they have attracted attention of authors of the present report to endogenous character of corrupt behavior of taxable subjects and probabilistic nature of audit and also for help in getting access to the appropriate papers[1].

1. Introduction

1.1. Preliminary remarks

The present report consists of three sections. The first section of the report represents introduction containing, in particular, statement of problem, literature review and description of some institutional features of taxation in Russian economy. In the second section the general equilibrium model with endogenous distribution of firms in terms of extent of tax evasion under uncertainty is considered. Sections 2.1-2.3 contain variants of the model for risk-neutral firms and sections 2.4-2.5 - for risk-averse firms. The third section is a conclusion. Appendix A contains symbolic notations. Appendix B is devoted to some characteristics of tax evasion in modern Russia.

The **object** of the present paper is to investigate tax evasion as consequences of decision making of taxable subjects. The **purpose** of the present project is to design the model of tax evasion under penalties and refusal of banks to grant loans, allowing evaluating economic losses produced by this kind of corruption.

1.2. Review of literature

The scales of tax evasion in the Russian economy are very high, thus the share of informal jobs on factory floors constitutes up to 20%, and the share of concealment of volumes of product up to 30-40% (see «Informal sector…», under the editorship of Dolgopiatova, 1998). According to the cautious evaluations the losses of the Russian budget and social funds caused by such behaviour constitute up to $5 billion annually. (Yakovlev, 1999a). Tax evasion is considered as a kind of corruptible behavior. Without deepening into economics of corruption we can mention that enormous literature is dedicated to the research of corruption on verbal and modeling level (see Levin, Tsiric (1998a, 1998b); Laffont and N'Guessan (1999); Levin and Satarov (2000)). As a rule tax evasion is considered in terms of the following problems:

[1] Authors of current paper want to acknowledge the help of officers of tax authority, tax police, businessmen and bank managers. Special thanks are also to our colleagues: E. Gurvich, S. Yitzhaki, E. Ershov and A. Yakovlev and others for their help in our research.

- Individual behavior of consumer or firm under possible punishment for tax evasion. Tax evader can spend various resources on either tax base reduction or bribing controlling authorities.
- Behaviour of tax authorities which duty is to control tax collection under condition of possible corruption. Tax authorities are acting under some constraint. In particular, these constraints include frequency of audit, effectiveness of audit depending on stuff level of proficiency, audit expenditures and so on.
- Government behaviour as a whole. Government is interested in tax collection as a source of income and spends some resources on tax collection and fighting tax evasion.

Basic literature on tax evasion is devoted to either some part of economic agent behavior or process and results of interaction of agents (games of the kind: tax evader and tax inspector audited him).

Corruption in tax system was considered in the works (Polterovich 1998), (Cremer and Gahvari, 1995), (Corchon, 1992, Hindriks J., Keen M., and Muthoo A. (1999), Myles G.D., 1995 - part III, 12) and others. In these works the conditions when corruption is profitable or not profitable for economic subjects, optimality of tax system and audit with different attitude towards risk are examined. In the works (Vasin, Agapova) and (Vasin, Panova) optimal organization of tax inspection ensuring the absence of corruption in the situation of Nash equilibrium is discussed. Schemes of tax evasion and the consequences of such behaviour of economic agents are considered in the works of Yakovlev (1999a,b), Wei (1997), Cowell (1990), Ghura (1998), Corchon, L.C., (1992), Hindriks J., Keen M., and Muthoo A., (1999). The endogenous nature of tax evasion is considered in J. Slemrod and S. Yitzhaki (1987, 2000) и J. Mayshar (1991). In this papers it is assumed that expenditures for control and fighting tax evasion depend on the size of tax collected. Especially we emphasize the paper, written by Gordon J. (1989), in which the moral aspects of individual and collective behaviour under tax evasion (in dynamics) are considered. However, numerous papers devoted to the problem of tax evasion practically do not touch upon a question of possible “economic” punishment of tax evaders, since another economic agents will not render them some services, for instance, banks will refuse to give them a loan. In our opinion, this aspect plays important role in making decision about tax evasion.

As a rule, econometric models based on macroeconomic hypotheses of relations among variable, or partial equilibrium models are constructed in order to investigate economic damage

caused by tax evasion. However we don't know any papers devoted to evaluation of damage produced by this kind of corruption that are based on general equilibrium models which allow to evaluate not only direct damage but also "full" damage. The latest constitutes object of our research.

Standard approach to the evaluation of economic damage caused by the certain existing at present phenomenon is in the following. It is supposed that an economic system possesses the function of public utility. The two states of economy are considered the real, in which the phenomenon being evaluated is present, and the imaginary, in which it is absent. The values of utility in these states will be generally speaking different. Suppose it is less in the first case. It is possible to evaluate the extent to which it is necessary to increase consumer earnings in this case so that the value of his or her purposive function should become the same as in the second. The amount of this compensatory income is the measurer of the damage caused by the phenomenon being evaluated. This approach was proposed in the work (J. Hicks, 1941) and developed in (P.A. Diamond , D.L. McFadden, 1974). J. Driffill, G. Mizon and A. Ulph (1990) have used it for the evaluation of economic losses because of inflation. To apply such an approach for numerical evaluation of damage caused by the certain economic phenomenon, it is necessary to be able to describe the economy in the hypothetical case of its absence as well. As a rule there is no information for this concerning the phenomena of a large scale. The use of the derivative of compensatory income by a certain parameter, measuring the scale of the phenomenon, is much more practical and no less informative. Let us call its value marginal costs or specific damage of the given phenomenon. Such an approach was used in the works (Ch. Ballard, J. Shoven, J. Walley, 1985), (Ch. Stuart, 1984) and (S. Movshovich, G. Krupenina, M. Bogdanova, 1997). Thus in the last work the proportion of marginal compensatory income (MCI) to the corresponding alteration of state expenditures, financed at the cost of tax revenues, estimates the value of tax burden of the population, created by the unit of governmental expenditures in the state of the economy being observed.

We use the analogous approach for the objectives of our research: the alteration of any of the parameters describing corruption will lead to the alteration of obtained values of purposive functions of all the participants and the necessity of compensation for each of them. In such approach, the damage caused by corruption is estimated by the sum of compensatory earnings necessary for all the subjects. The proposed methods are close to the methods used in the work (Movshovich S.M., Krupenina G.A., 2000).

We use increment value of the compensating income and state expenditures under changing of various exogenous parameters for estimation of losses. Their relative difference

determines the size of marginal losses under unit change of parameter. Their relative difference determines the size of marginal losses under unit changing of parameter. For more complete examine of probabilistic effects we take into consideration recommendations stated in E. Koskela (1983), J. Stennek (1999).

1.3. Institutional specifics of Russian tax evasion

The decision-making by firms on tax evasion is determined by the following circumstances: the firm decides 1) whether to conceal part of the profit or not, and how much to conceal in the former case; 2) whether to take the credit in bank or not, and how much to borrow in the former case, according to the principle of profit maximization. This profit is understood as a "profit" under conditions of probability being caught and punished for tax evasion. Possibly, the firm tax evasion activity needs certain expenses, such as bribes, payment for advisers services etc.. In the current section for simplicity we omit these expenses: their inclusion slightly complicates the model, but not essential for the model results. It is assumed that if the firm conceals some share of the profit and this fact is detected by bank, then the firm will be discredited. If tax authority detects the fact of concealment, the firm will be punished. Thus, firm's decision is determined as a consequence (or part) of the equilibrium of economic system consisting of set of firms, representative consumer, banking system and tax bodies. In this paper as distinct from J. Mayshar (1991), J. Shemrod and Sh. Yitzhaki (1987, 2000) it is not assumed that the financing and, therefore, activity of tax bodies is greatly determined by volume of taxes collected. We assume, that the probability of concealment detection is a fixed parameter of the model (identification of dependence of detection probability and of tax bodies activity on volumes of the collected taxes is practically impossible under Russian conditions).

The firms' behavior under risk is considered for risk-neutral and risk-averse firms. In particular, the papers of E. Koskela (1983), J. Stennek (1999) are pointed out the importance of taking into account latter feature.

Thus, in this section firm's policy on tax evasion has an endogenous character.

The fact, that firm's tax evasion policy greatly depends on "behavior" of tax bodies, is obvious. Therefore let's explain interrelation of banks credit policy and behavior of firms - borrowers in terms of tax evasion.

We made in-depth interviews with a number of experts: employees of consulting firms, businessmen and bankers.

Experts' undivided opinion is that at the present time loans play very important role for Russian firms especially for middle-sized and large. According to the experts (bankers) after

1998-year crisis many banks get their profit from domestic firms crediting. At the same time, more than 50-70 % of firms try to obtain short- or long-term loans. For this purpose the large firms seek after a positive credit history. As to middle-sized firms they frequently have no credit history. But all kinds of firms aspire to demonstrate to bank “good-looking” financial reports. Their reputation plays the important role in credits granting. Systematical firm audit supports firm’s reputation for banks. As to small firms, they, as a rule, have no credit history, and their audit is carried out irregularly, or is not carried out at all. The banks, making their decision on granting loans to the firms (both large, middle, and small), emphasize interrelation of firms with the budget: to what extent the firm is “clean" before tax bodies. For this purpose the bank investigates in details financial reports of firm and on their basis comes to a conclusion on the risk that the firm is the debtor or can become a debtor in respect of the budget. The point is that according to the Civil Code of Russian Federation, in the case of conflict between the firm and the bank because of the non-returned loan, first of all the firm must pay all taxes and only after it the firm has to pay back the bank loan. At the present time according to the Moscow commercial banks information, up to 20% of negative response in granting the loan occurs because of the firm tax troubles.

Nevertheless, the level of tax avoidance and tax evasion in Russian economy is very high. Experts (representatives of tax police) point out, that firms have learned that the money credit can be obtained only if they have a proper look in the face of tax bodies and banks. "Price" of a "proper" look makes up from 5 to 40 % from the amount that the firm should pay as the taxes. The part of this sum is the cost of audit, the cost of designing more or less complex schemes of tax avoidance and partly - for bribing, that is usually used by small firms.

Remark 1.2: Let’s note the following interesting circumstance. The bribes are some alternative to expenses on putting in order of the complex documentation and creations of the schemes tax evasion. The alternative, the "brains" against bribes, is appeared. "Brains" – to the construction of the complex schemes of transforming tax evasion to tax avoidance and putting in “a good” order of the financial documents. According to the experts, “good brains” are cheaper than bribes but often it is more difficult to find them”. Moreover, the effective use of "brains" is possible only if the top managers and/or the firm owners cooperate with them. Apparently, the role of bribing for tax evasion decreases with the growing of firms, but the expenses for “financial proper look” increase. Bribes can also be used as a means of competitive struggle, "setting" tax bodies against the competitors. In our model such features of firm behavior is not considered.

2. Tax evasion as endogenous choice of firms

2.1. Equilibrium in the model of tax evasion

To evaluate damaged caused by tax evasion **the general equilibrium model** is used.

2.1.1. Description of the model

The model includes three types of agents: generalized consumer, generalized producers – firms and government. Consumer maximizes his utility function under appropriate budget constrains. Firms maximize their profit. The government collects taxes. The main active agent evaded taxes completely or partly or paid all taxes is a firm. In the present model it is assumed that firm making decision to what extent evade taxes chooses optimal policy in terms of objective function. Firm's objective function takes into account the probability of "punishment" for tax evasion and its "weight". It is assumed that firms differentiate one from another by production possibilities, in other words, by productivity. And this difference affects the decision on tax evasion made by the firm. In the present paper two variants of the model is considered: first – for risk-neutral firms and second – for risk-averse firms. The model is assumed to be one – product: al the firms produce the same product, which is consumed by consumer. Two periods is considered: in the first period firms make decision on the size of output and the size of loan. In the model bank sphere is not considered explicitly, but it is the part of institutional environment, in which firms act. The bank (generalized) grant firm a loan if it has enough information to be quite sure that the loan will be refunded. It is assumed that the loan the firm is used for expansion of production in the second period and hence the firm makes decision on borrowing in terms of profit maximization for the two periods.

It is assumes that all the firms are divided into three groups in terms of law-abidance. The first type; firms paid all taxes – "honest" firms. The second type: firms evaded taxes completely, i.e. firms that do not pay taxes at all – "dishonest firms. The third type constitutes firms that pay taxes partly – "semi-honest" firms. It is important to note that the type firm is fallen is determined only by its rational choice depending on the state of the system as a whole, i.e. grouping into types of law-abidance is one of "endogenous" parameter of equilibrium state of the economic system as a whole.

So, the economy is modeled by aggregate deterministic static but two-period equilibrium model of Arrow-Debreu type that includes producers, consumers, government and bank system. Variety of products and services is aggregated into one product as well as all the kinds of labor are aggregated into one kind of labor. The production sphere consists of the set of infinitesimal

producers and each of them has certain productivity - “effectiveness” of labor and capital transformation into consumer product.

So, production sphere includes a set of the producers. They are described by parameter t, $t \in (0,1)$, and producers t, $t+dt$ during one period can upgrade production on the basis of project which provides increase of efficiency $f(t)$ under investment $x(t)$. Actual investments x are equal to 0, or $x(t)$. Remaining producer’s characteristics do not depend on t. In particular, producers t, $t+dt$ use a part of total production funds kdt and release an appropriate part of the product $F(l)dt$, where F - production function with fixed k, l is a quantity of labor. If producer declares all incomes, then his profit is determined. If he conceals a part of incomes, then he finds himself in risk conditions. Risk is generated by possible audit revealing the concealment with unit probability. In the model the revelation of fraud entails two types of consequences having an influence upon efficiency of his activity. Firstly, an enterprise is levied underpaying taxes and fined. Secondly, an enterprise can be concerned with bank credits for modernization and extension of production. Since the market of credits, as a rule, is in non-equilibrium then enterprises are forced to prove their solvency when appeal to financial institutions for credits. We suppose that as soon as dishonesty of financial activity of enterprise is discovered its credit reputation is called in question and it is discredit. Even if the audit isn’t carried out the proof of solvency requires high enough profitability of current activity. If taxing authority and financial institutions use the identical information of incomes, then producers cannot conceal arbitrary large part of profit from taxation. Thus, even if fines are insufficient for deterrence the producer from the fraud, his tendency to modernization will force him to decide whether it is worthy to conceal incomes and to what extent.

2.1.2. Producer’s behavior

Let's assume, that the producer is risk-neutral, i.e. he maximizes expected discounted profit (suggested approach is easily generalized to the case of risk aversion) i.e. at arbitrarily given t he solves the following problem:

$$(1-P)((1-a)(F(l)-wl-h)+h+\delta/(1-\delta)((1-a)(F(l)-wl+f(t)x)-rx)+$$

$$P((1-a)(F(l)-wl)-sh+\delta/(1-\delta)((1-a)(F(l)-wl))) - max \qquad (2.1)$$

on l,h,x , $if\ h>0$ and

$$(1-a)(F(l)-wl)+\delta/(1-\delta)((1-a)(F(l)-wl+f(t)x)-rx) \text{ -- } max \qquad (2.2)$$

on $l, x,$

$$if\ h=0. \qquad (2.3)$$

Here w - wages for a unit of labor, h a part of the profit concealed from the taxation, a- the rate of profits tax, δ- discount, $f(t)$ - productivity of new investments in production, x - size of these investments, r - percent for the credit, P - probability of audit of income declared by producer, s - fine income concealment, k-production funds. In construction (2.1) we suppose that for each t there is a project raising productivity and that the project application does not change employment, but increases the output. We assume that hereinafter both demand for labor and wages selected in the first period remains constant; the application of the new project requires one unit of time; investments are carried out at the expense of credits solely; expenditures and outputs of the second period keep the same in subsequent periods. Maximum of l is reached at all t under $F(l) - wl - max$. Let's denote maximum point by l^*.

Construction (2.1) implies that producers do not evade taxes, if $(1-P)a-Ps < 0$, i.e. the probability of audit and fine size are sufficient to provide complete tax payments. Hereinafter we suppose that an opposite inequality holds. Producer is concerned with investments, if $(1-a)f(t) > r$. This inequality is also supposed to be true. Let's also assume that producers study interests (2.1) - (2.3) only, i.e. there are no lawful producers contrary to (2.1) - (2.3) among them.

Let's show that in view of (2.1) - (2.3) producers are subdivided into three types; each type has its own value of parameter t. If producer wants to get the credit for sure, i.e. $h=0$, then his profit will be equal to

$$R(1,t)=(1+ \delta/1-\delta)((1-\alpha) (F(l^*)-wl^*)+ (\delta/1-\delta)((1-\alpha)f(t)-r)x.$$

If producer neglects investments nourish the hope to get more benefit from tax evasion, then from (2.1) follows that he will get

$$R(2,t)= (1+ \delta/1-\delta)(1-\alpha) (F(l^*)-wl^*) + ((1-P)a-Ps)(1-\alpha) (F(l^*)-wl^*).$$

Here the second term is an expected profit from tax evasion. At last, he will get $R(3,t)$ if $h>0$.

Producer belongs to the *first type*: declares true profit and pays all taxes, if

$$R(1)>= max(R(2),R(3)). \quad (2.4)$$

Producer belongs to the *second type*: does not pay taxes at all, if

$$R(2)>= max(R(1),R(3)). \quad (2.5)$$

Producer belongs to the *third type:* pays taxes partly, if

$$R(3)>= max(R(1),R(2)). \quad (2.6)$$

Let's denote the share of producers of type i, i=1,2,3, determined by one of the conditions (2.4) - (2.6) by $\mu(i)$, $\Sigma\mu(i)=1$. Model (2.1) - (2.6) reflects the following types of producers' behavior concerning concealment of a part of incomes from the taxation. Producers (there are $\mu(1)$ producers of this type, $t\in T(1)$) declare true incomes, if either the fine is large and the audit is frequently conducted, or condition (2.4) is true. If producer either is not concerned with investments or condition (2.5) is true, then he evades taxes completely (there are $\mu(2)$ producers of this type, $t\in T(2)$). If condition (2.6) is true (there are $\mu(3)$ producers of this type, $t\in T(3)$) then he pays a part of tax sum. Shares $\mu(i)$ vary together with $T(i)$ under change of audit parameters, tax rates, wages and percent.

Remark 2.1.2: We'd like to stress the significance of R values, depending on exogenous parameters of the system and equilibrium. Using R values introduced in the present paper each individual firm unambiguously classified by the type of behavior and at the same time it allows unambiguous determination of the share of firms that fall into particular category.

2.1.3. Consumer's behavior

Consumer in the model maximizes utility function u depending on the volume of consumption and free time. Equilibrium models including one representative consumer bypass matters of income allocation, wages differentiation and its influence on individual labor efforts. This approach we extend over lawful consumers and consumers who evade taxes completely. Instead of splitting consumers into groups similarly to that made with producers we shall consider the joint representative consumer, whose income is made up of the two parts: income of law-abiding people and income of people evading taxes. Incomes are formed by wage and a part of profit residual left after payment of tax and fines. Let's denote this part by ρ. Let ξ be the share of law-abiding consumers, $1-\xi$, - the share of consumers evading taxes, l - gross supply of labor, τ - income tax rate. Then l and the volume of consumption c are found from the solution of the problem

$$u(c, L-l)\Rightarrow\max \tag{2.7}$$

subject to

$$c = (1-\tau\xi))(wl+ \rho((F(l^*)-wl^*)(1-\alpha(\mu(1)+(\mu(2)+\mu(3))P(1+s))+$$

$$+\alpha\mu(3)rk/1-\alpha) \tag{2.8}$$

2.1.4. Government's behavior

The government collects taxes, which together with fines are the only source of financing of its expenditures. Expenditures are made up of procurement of product, g, ensuring the needs

of the state and not influencing the utility of the representative consumer. State budget is supposed to be balanced, i.e. we get

$$g = \xi\tau wl + \mu(1)\alpha(F(l^*) - wl^*) + (\mu(2)+\mu(3))P(1+s)\alpha(F(l^*)-$$

$$-wl^*)+\mu(3)\alpha rk/1-\alpha \qquad (2.9)$$

Suppose that undisposed profit and appropriate part of the product is removed, without increase neither consumer's utility, nor payload on the system. If labor is also balanced

$$l=l^*, \qquad (2.10)$$

then the economy is in the equilibrium state in the first period (inasmuch as we assume that capitals market is in non-equilibrium, the interest rate for credits is supposed to be given and the capital balance is not considered). Since the future we consider only in that measure, that is necessary for our purposes, the equilibrium in the future periods is not considered.

Since we concern with the future only to the extent of necessity for our purposes, the equilibrium in the future periods is not considered.

2.2. Changing of producers division into types of tax evasion under changing economic system parameters

From (2.8) and (2.9) the role of division of producers into types in the analysis of influence of economic parameters on public losses is clear. The main problem of this section is investigation of influence of these parameters on producer's choice of one of the three considered above types of behavior. Consider (2.4) - (2.6), defining fragmentation of a segment [0,1] into *T (i)* sets having length of μ *(i)*, and by that we shall revel what and how influences enterprise's choice of its taxation policy. Let's formulate definitions of the profit *R (i, t)*. Let

$$A = (1 + d/1-d)\ (1-a)\ (F\ (l\ ^*)-wl\ ^*),$$

$$B\ (t) = (d/1-d)\ ((1-a)\ f\ (t)\ -r)\ x\ (t),$$

$$C = ((1-P)\ a-Ps)\ (F\ (l\ ^*)-wl\ ^*).$$

Then

$$R(1,t)= A + B(t),$$

$$R(2,t)=A + C,$$

$$R(3,t) = A + C +(1-P)B(t) - rk/1-a).$$

Suppose for distinctness that derivative $B'(t)<0$, $B(0)>C> B(1)$.

Then $R(1,t)>R(2,t)$ under $t<t(12)$.

If $t>t(12)$ the opposite inequality holds. Since $R(3,t)=R(2,t)+(1-P)B(t)-rk/1-a)$, if shares of both types is not equal to 0, then

$R(3,t)>R(2,t)$ under $t<t(32)$.

If $t>t(32)$ the opposite inequality holds. Since $R(3,t)=R(1,t)-PB(t)+C-rk/1-a)$, if both types shares are not equal to 0, then

$R(3,t)>R(1,t)$ under $t>t(13)$.

Under $t<t(13)$ the opposite inequality holds.

Thus, if $\mu(i)>0$ for all i, then T(1) lies at the left of T(3), a T(3) - at the left of T(2), i.e. these sets are divided by points t(13) u t(32).

It implies that change of shares $\mu(1)$ and $\mu(3)$ is determined by movement of the point $t(13)$: $B(t)-C/P+rk/(1-a)P=0$ under change of economic parameters and shift from $\mu(3)$ to $\mu(2)$ and vice versa – by movement of the point $t(2)$: $B(t)-rk/1-a)(1-P)=0$. Note here that, from $B'(t)<0$ follows that $B(t(32))< B(t(13))$, i.e. $(1-a)C>rk/(1-P)$ and thereby assumption on existence of 3 types of tax evasion requires relatively high productivity of available capital.

Let's consider as illustration $B(t)$ function of the following type: $B(t)=(\delta/1-\delta)((1-\alpha)f(0)-r)x(0)(q-vt)$, where $q, v>0$ – appropriate constants. Since A, C depend on parameters α, P, s, r, w, then shifting of $t(13), t(32)$ and thereby $\mu(i)$ values is determined by these parameters change. For instance, if the audit probability changes then change of $\mu(1)$ is determined by equation $-(\delta/1-\delta)((1-\alpha)f(0)-r)x(0)vdt(13)+dP\{(\alpha+s)(F(l^*)-wl^*)/P-((1-P)a-Ps)(F(l^*)-wl^*)/P^2-rk/(1-\alpha)P^2\}=0$, where $dt(13)=d\mu(1)$. Similarly, $d\mu(2)=-dt(32)$, $d\mu(3)=-dt(13)+dt(32)$.

2.3. Equilibrium state of economic system and public loss

2.3.1. Equilibrium

The Equilibrium state is determined by the conditions of optimal solutions made by producers and representative consumer, and also by conditions of balance of state budget and balance in the labor market. Firstly, consider the case when $\rho=0$. Let's write out these conditions

$$F'(l^*) = w$$

$$u_2(c, L-l) = u_1(c, L-l)(1-\tau\xi)w \qquad (2.11)$$

$$c = (1-\tau\xi)wl,$$

$$l = l^*,$$

$$g = \xi\tau wl + \alpha(F(l^*) - wl^*)(1-(\mu(2)+\mu(3))(1-P(1+s))+\mu(3)\alpha rk/1-\alpha$$

The system (2.11) determines $c, l, l^* \omega, g$ as a function of parameters $z=(\mu, \xi, P,s,r,\alpha,\tau)$.

Influence of any of parameters on public loss we shall measure by the value of Marginal Excess Burden

$$MEB(z) = (de/dz - dg/dz)z/g, \quad (2.12)$$

where de – - increment of the compensated income.

Determined in such way Marginal Excess Burden is an analogue to elasticity. In particular, if increment of the compensated income $de=0$ then Marginal Excess Burden determined according to (2.12),

$$MEB(z) = (- dg/dz)z/g,$$

is elasticity of taxes collected (government revenue) with respect to corresponding parameter. If the parameter is probability being caught for tax evasion then we get elasticity of tax collected with respect to probability in equilibrium state. Thus, we can evaluate audit effectiveness. Similarly effectiveness of other system parameters can be evaluated.

Remark 2.3.1: In some papers on tax evasion (Mayshar, J. (1991), Slemrod, J. and Ytzhaki, S. (2000)) the effectiveness of expenditures on audit is analyzed. In our paper for this purpose MEB can be used as function of audit expenditures, q, which in turn determine audit probability, $P=P(q)$.

If we know this function of expenditures influence on probability of detection then we can represent MEB as the following: $(-\partial g/\partial P)(\partial P/\partial q)g/q$. It is clear that the effectiveness of expenditures is greatly determined by elasticity of function.

2.3.2. Comparative statics equations under compensation of consumer's utility

To determine derivatives in (2.12) we consider comparative static equations under compensation of consumer's utility

$$u_1(c, L-l)dc - u_2(c, L-l)dl = 0;$$

$$F''(l)dl = dw;$$

$$u_{21}(c, L-l)dc - u_{22}(c, L-l)dl =(u_{11}(c, L-l)dc - u_{12}(c, L-l)dl) (1-\tau\xi)w + u_1(c,L-l)((1-\tau\xi)dw - \xi w d\tau - \tau w d\xi);$$

$$dc = de + (1-\tau\xi)ldw + (1-\tau\xi)wdl - wl\tau d\xi - wl\xi d\tau; \quad (2.13)$$

$$dg = d(\xi\tau wl) + (F(l^*) - wl^*)(1 - (\mu(2)+\mu(3))(1-P(1+s))d\alpha - \alpha l(1 - (\mu(2)+\mu(3))(1-P(1+s))dw - \alpha(F(l^*) - wl^*)(1-P(1+s))(d\mu(2)+ d\mu(3)) +$$

$\alpha(F(l^*) - wl^*)(\mu(2)+\mu(3))\ ((1+s)dP + Pds) + \alpha rkd\mu(3)/1-\alpha + \mu(3)\alpha kdr/1-\alpha +$

$\mu(3)rkd\alpha/(1-\alpha)^2$

In particular, when there are changes of audit parameters and rate of percent only $dc = dl = dw = de = 0$, $dg = \alpha(F(l^*) - wl^*)(1-P(1+s))d\mu(1) +$

$\alpha(F(l^*) - wl^*)(\mu(2)+\mu(3))\ ((1+s)dP + Pds) + \alpha rkd\mu(3)/1-\alpha + \mu(3)\alpha kdr/1-\alpha.$

2.4. The influence of uncertainty on behavior of risk-averse producers

So far we supposed that firms are risk-neutral, i.e. their objection is to maximize expected income. Primarily for simplicity we evaluate this influence using simplified version of the model and assume that firm's tax liabilities are fixed and equal to T and income is equal to I. A firm chooses to declare an amount T', $0 \geq T' \geq T$. One of the two outcomes is possible: either the declaration is not chooses to be checked and a firm pays T', or the firm is audited and pays $T+s(T-T')$. The probability of audit and therefor the probability of the second outcome is equal to P. Risk-neutral firm maximizes its average income, i.e. the function

$I-(1-P)T'-P(T+s(T-T'))=I-TP(1+s)-T'(1-P-Ps) \Rightarrow \max$

Obviously, $T'=T$ with $P>1/(1+s)$ and $T'=0$ otherwise.

Now let's consider behavior of risk-averse firm.

A). Following Von Neumann-Morgenshtern, suppose that concave utility function of income $U(x)$ is given and a firm aims at maximizing its expected utility, so its objective is

$U=U(I-T')(1-P)+U(I-T(1+s)+sT'))P \Rightarrow \max$

Let U is a smooth increasing function. Then the necessary condition for maximum is

$-U'(I-T')(1-P)+sPU'(I-T(1+s)+sT')=0,$ (2.14)

It's known (Arrow) that if the measure of risk-aversion is not changed with changing of U argument then U is exponential unction, that means $U=-exp(-ax)$. Let $H=T-T'$, $a=T$, $x(1)=H$, $x(2)=-sH$, then from (2.14) we obtain $H=max\{0, min\{-1/T(1+s)\ \ln Ps/(1-P), 1\}\}$. It is clear that $H=0$ when $P<1/1+s$, as in the case of risk-neutral firm.

B). Following Markowitz, suppose that firm determines the declaration amount maximizing a function which is increasing in average income and decreasing in its dispersion. In the primitive case $f=M-cD$, i.e. firm solves the following problem:

$(1-P)H-PsH-c/2((1-P)(H-(1-P)H+PsH)^2+P(-sH--(1-P)H+PsH)^2)) \Rightarrow \max$

With $c=1/T$ it implies that

$H=max\{0, min\{T(1-P(1+s))/(1+s)^2P(1-P),T\}\}$

Further we'll evaluate what deterministic income is achievable in the given lottery under various income losses compensation mechanisms and what is the scale of tax evasion in these cases.

C). What level of permanent income would be insured if income is insured? Let the firm aims at permanent income x, i.e. in the second case it claims to insurance compensation at the rate of $x-(I-(T+s(T-T'))$. At that in the first case it pays insurance premium $I-T'-x$. The difference between expected premium and expected compensation $(1-P)(I-T'-x)-P(x-(I-(T+s(T-T'))$ is insurer's expected income $i(y)$, $y= x-(I-(T+s(T-T'))$. It is non-negative and increases in y with decreasing rate. A firm gets a permanent income x, if

$$x=I-(1-P)T'-P(T+s(T-T')-i(x-(I-(T+s(T-T')), \quad (2.15)$$

Let's find the maximum amount of x in T'. If $i=0$, then we return to the case of risk-neutral firm. If $i(y)=i>0$, then the firm chooses $T'=T$ with $P>(1-is)/(1+s)$ and $T'=0$ otherwise, i.e. the amount $s'=s(1+i)/1-is$ can be considered as discounted penalty. In general, equation (2.15) determines implicit function $x(T')$. In the maximum point its derivative $dx/dT'=(1-P(1+s)+si'(y))/(1+i'(y))$, $y= x-(I-(T+s(T-T'))$ is equal to 0. For example, if $i(y)=iy-ay^2$, then $i'=i-2ay$ and requirements on function i are met if $y<i/2a$, i.e. $a<i/2(T+s(T-T'))$. In this case T', x are determined by the following system:

$1-P(1+s)+si'(y)=1-P(1+s)+s(i-2a(x-(I-(T+s(T-T'))=0,$

$x=I-(1-P)T'-P(T+s(T-T')-i(x-(I-(T+s(T-T'))-a\ (x-(I-(T+s(T-T'))^2.$

D). Let in the first case firm deposits money at a bank at rate r_1. In the second case it takes a loan in the sum of $x-(I-(T+s(T-T'))$ at rate r_2, $r_2 > r_1$. Firm's bank account is balanced, if $(1-P)(1+r_1)(I-T'-x)=P(1+r_2)(x-I+ (T+ s(T-T'))$. Now $T'=T$ with $(1-P)(1+ r_1)-P(1+ r_2)s<0$, i.e. $P>1/((1+ r_2)s/(1+ r_1)+1)$. The expression $(1+ r_2)s/(1+ r_1)$ can be called discounted penalty.

2.5. Tax evasion in equilibrium state

Let's now return to the analysis of the model (2.1)-(2.3). First of all we bring the following designations in: let $I=F(l^*)-wl^*$, $I(1)= \delta/(1-\delta)((1-a)I$, $I(0)=I-rk/1-a$, $V=\delta/(1-\delta)((1-a)f(x)x-r(x))$. Notice, that all these values depend on t. If producer conceals part of profit h from taxation, then his discounted profit R is equal to

$R(1)=(1-a)I+I(1)+V,$ if $h=0$.

$R(2)=(1-a)(I-h)+h+I(1)+V,$ if $0<h<I(0)$ and his is not audited,

$R(2)=(1-a)I-sh+I(1)$, if $0<h<I(0)$ and he is audited,

$R(3)=(1-a)(I-h)+h+I(1)$, if $I(0)<h<I$ and his is not audited,

$R(3)=(1-a)I-sh+I(1)$, if $I(0)<h<I$ and he is audited.

Recall that producer is audited with probability P and increase of profit V is produced by investments. If selection of optimal behavior is restricted to the approaches B)-D), then its results don't depend on constant part of the profit $(1-a)I+I(1)$. So we omit it and thus choice of h is determined by the following information of possible outcomes.

If firm chooses $h=0$, then with probability 1 it gets a rise V.

If firm chooses $0<h<I(0)$, then with probability $1-P$ it gets $V+ah$ and with probability P it gets $-sh$.

If firm chooses $I(0)<h<I$, then with probability $1-P$ it gets ah and with probability P it gets $-sh$.

Under above stated approaches C) and D) results hold true if penalty is replaced by discounted one, calculated in C) and D) respectively. The average of utility distribution is equal to

V, if $h=0$

$(1-P)(V+ah)-Ps'h$, if $0<h<I(0)$

$(1-P)ah-Ps'h$, if $I(0)<h<I$.

If $(1-P)a-Ps'<0$, then firm doesn't conceal its profit, $h=0$. If $(1-P)a-Ps'>0$, either $h=0$, or $h=I(0)$, or $h=I$, depending on which of the following is greater V, $(1-P)V+((1-P)a-Ps')I(0)$, and $((1-P)a-Ps')I$.

If approach B) is used and $(1-P)a-Ps'<0$, then firm doesn't conceal its profit too, $h=0$. If $(1-P)a-Ps'>0$, then firm solves the following problems

$R(2,h(2))=$

$\max\{(1-P)(V+ah)-Psh-c[(P^2(V+ah+sh)^2(1-P)+P(1-P)^2(sh-V-ah)^2], 0\le h\le I(0)\}$.

$R(3,h(3))=\max\{(1-P)ah-Psh-c[P^2(1-P)(ah+sh)^2)+P(1-P)^2(sh-ah)^2], I(0)\le h\le I\}$

and finds $\max\{V,R(2,h(2)),R(3,h(3))\}$. Firm conceals from taxation that amount h, i.e. either 0, or $h(2)$, or $h(3)$, which gives this maximum.

3. Conclusion

In this paper we introduce general equilibrium model, taking into account such phenomenon as tax evasion of producers and consumers. Mainly, we focus on production sector which is divided into three sub-sectors according to the extent of tax evasion: firms that pay all taxes, firms evaded taxes completely, firms that pay taxes partly. The division of firms into types in terms of law-abidance is determined endogenously and depends on effectiveness of firm's activity. Characteristic feature of the model that differs our paper from other known papers on tax evasion is that firm's decision on tax evasion is determined not only by punishment for violation of law but also by the fact that banks refuse to grant loans to firms-evaders. In our opinion, and many experts on Russian economy agree with it, the latter factor forces majority of firms to pay taxes, completely or partly.

In the paper (section 2.3) we represent first order conditions characterizing equilibrium state of economic system, including firms classification according to types of tax evasion and hence allowing evaluating equilibrium shares of each kind of firms in dependence on exogenous parameters, in particular, audit probability. Also we show how audit expenditures can be taken into account in the framework of our model (remark 2.3.1).

Two kinds of models characterizing firms' behavior under uncertainty associated with possible audit are designed in the paper. The first kind – is equilibrium with risk-neutral firms (section 2.1.2), the second kind - is equilibrium with risk-averse firms (section 2.4).

To evaluate damage caused by tax evasion we use such index as MEB allowing to evaluate dependence of equilibrium integral damage on some exogenous parameters, in particular, on audit probability, penalties and so on (section 2.3.1). In order to use this index equation system characterizing comparative statics of equilibria (section 2.3.2) and allowing to compare the influence of exogenous parameters changes on endogenous variables in terms of small variations is represented. If we know these changes we can calculate integral index MEB evaluating damage.

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Appendix A: Symbolic notation

Tax-evasion characteristics:

μ – the share of law-abiding producers

ξ - the share of law-abiding households

Tax rates:

α - profit tax rate

β - value-added tax rate

γ - charges on the wages fund

τ - income tax rate

Producer:

p – price of product ($p=1$)

w – wages for a unit of labor

$x(t)$ – investments volume

$f(t)$ – productivity (increase of efficiency) of investments

h – a profit share concealed from taxation

δ - discount

r – percent for credit

P – audit probability

s – fine for profit concealment

k – production funds

Consumer:

l - gross supply of labor

L - – overall time spent on work and leisure (excepting time of dream)

c - the volume of consumption

Government:

g – government expenditures

Appendix B: Some characteristics of tax evasion in Russian economy

For purpose of estimation of corruptible relations in the case of tax evasion in the Russian economy, importance of which stresses in many papers we

- analyzed scientific and publicist publications on the given subject
- made series of in-depth interviews with the experts (total number of interviews is 22):

1. Managers and owners of firms
2. Managers and owners of banks
3. Accountants
4. Taxation consultants
5. Representatives of tax inspection and representatives of tax police

Data, received from experts, supplement the information drawn from the publications.

Summarizing experts' judgements and analysis of the publications we shall indicate the following:

1). Not less than 80 % of firms evade from tax payments. The scale of evasion constitutes from 20 % up to 90 % of total tax liabilities.

2). Tax evasion takes the most significant forms for big firms, if divide, conditional to the size of the tax liabilities, all firms on small-sized, medium and big. Evasion by small firms is less than the big firms but more then the medium. The logic of this fact may be explained by the model of tax evasion close to the model Hindriks, Keen and Muthoo (1999) and is confirmed by the research of A. Yakovlev (unpublished speech at the meeting in the Center of Economic Reforms, April 2000). The main cause of these facts is the opportunity and capacity of firms to create the complex schemes for distortion of information.

The analysis of the publications and the interviews shows that corruptible collusion by means of "direct bribery " (which is the main point of many papers on corruption in tax authorities) is not substantial and pervasive for the Russian economy. Usage less or more complex schemes of distortion of business - information is specific and typical for Russian tax evasion system.

Distribution of the ways for tax evasion corruption is the following (percentage of volume of taxes evaded):

- a direct payoff ("bribe") to tax authority - 3-7 %

- direct distortion of the information in documents - 3-7 %
- design and usage of concealment schemes - 80-90 %.

The costs on tax evasion taxes ("transaction costs") are not high and may be evaluated as 5 % - 40 % of tax evaded sums. Hereinafter, mentioning about the value of bribes first of all we will mean this kind of expenditures for distortion of the information and for design of fictitious firms and so on.

The institutional data on the illegal activity and some quantitative characteristics may be found in the following tables.

	Schemes of tax evasion		
	"The firm-internal scheme "		"External scheme"
	version A	version B	
Basic schemes of enterprises' means transference into the shadow turnover			The main feature of this scheme is a transportation of means of the given enterprise into account of another legally operating firm, implementing within the framework of actual economic operations with this firm
1. Underestimation (concealment) of the enterprise's income level	1) Direct withdrawal of a part of proceeds received by the enterprise in the cash form from realization, with consequent use it on wages payment, covering of certain material expenditures and unaccounted profit formation etc.		Delivery of production on a undercharge (versions - delivery of production without prepayment and with a significant payment postponement; the acceptance of promissory note at nominal price as payment for delivered production under condition that at the market this promissionary note is sold with significant discount; making of export operations with partial or full "nonreturn" of means into Russian accounts; Purchases on an overcharge (versions - prepayment with delay of physical delivery of goods; payment on the import contracts not accompanied by recipience of actual commodity in Russia; purchase of production on barter with unprofitable proportions of exchange and etc.)

	2) Realization almost all production for unaccounted cash (for example, in clothes or wholesale provision markets).		
2. Artificial overestimate of expenditures		Artificial overestimation of material inputs by means of payment for the fictitious agreements	Purchases on an overcharge (versions - prepayment with delay of physical delivery of goods; payment on the import contracts not accompanied by recipience of actual commodity in Russia; purchase of production on barter with unprofitable proportions of exchange and etc.)
Firm's characterization	Small firms of retail trade and services rendering or production and wholesalers firms, independently realizing their production to the population. High level of information distribution about illegal activity among employees.	Enterprises, which have no access to the commodity markets. Low level of information distribution about illegal activity among employees.	This scheme requires trusting and relaxed relations of the given firm and firm-counteragent or direct control over activity firm-counteragent. Medium and large enterprises realize it.
Procedure	"reverse encashment"	"encashment"	
Commission payments to intermediary consulting firms for encashment/ reverse encashment	1-2%	5%	

Services rendered by specialized consulting firms to evaders

Services	Payments
1. Replacement of founders: a) a simple scheme (that is new "founder" becomes a person of no fixed abode or lost passport) b) complex scheme (in two stages: replacement of the present director for new one, and then sale of the	$200-500

corporation to the new founder who is not being a citizen of Russian Federation)	
2. Merging or joining to the sham firm.	$500-800 (+5-10% of the indebtedness), with juridical support during one year after joining - $1.5-2.5 (+5-10 % from the indebtedness)
3. Official liquidation with removal from the accounting at taxing authority and registration chamber	$1200
4. Bankruptcy with preliminary transference of firms assets.	$1000 (+10-15% of the sum от суммы "rescued" assets

Thus, at least 90% of overall tax evasion activity is connected with development and implementation of the complex schemes of information distortion rather than with “bribery" of tax inspectors or tax police. In experts opinion the scale of tax evasion in Russia constitutes up to 70 % for the juridical person and about 30 % - for the natural persons. A share of expenditures spent on tax evasion, which can be treated as "bribe", constitutes 2-5 % conditional to the level of evasion.

In conclusion of this section we’d like to note that after flat income rate fixation equal to 13% in 2001 the problem of tax evasion by consumers to a great extent ceases to be relevant for Russian economy.